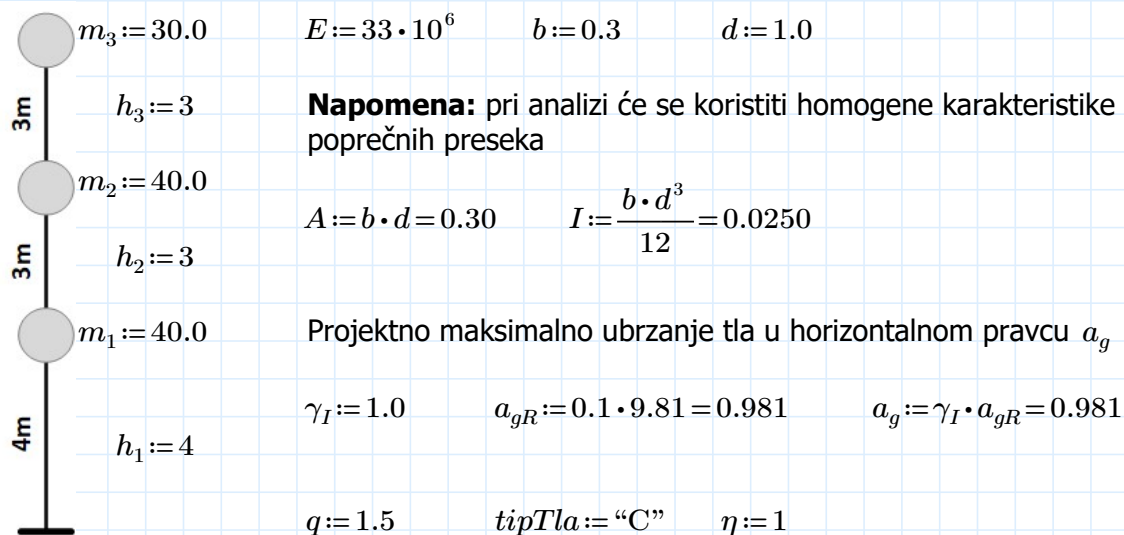


## Primer-MDOF-Spektralna-analiza-odgovora

**Podaci:**  $E = 33 \text{ GPa}$ ,  $b/d = 30/100 \text{ cm}$ ,  $m_1 = 40 \text{ t}$ ,  $m_2 = 40 \text{ t}$ ,  $m_3 = 30 \text{ t}$ , referentno maksimalno ubrzanje tla u horizontalnom pravcu  $a_{gR} = 0,1g$ , faktor ponašanja  $q = 1.5$ , tip tla C, tip elastičnog odgovora 1, faktor značaja  $\gamma_I = 1.0$ , relativno prigušenje 5%.

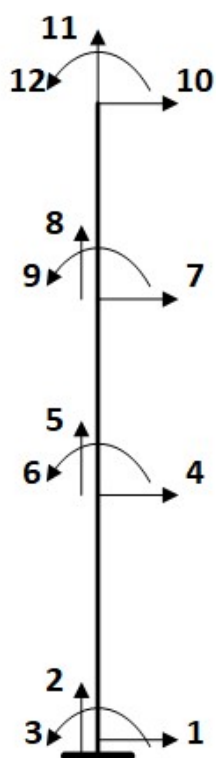


## Statički i dinamički model

**Napomena:** analizira se samo mogućnost vibracija u horizontalnom pravcu

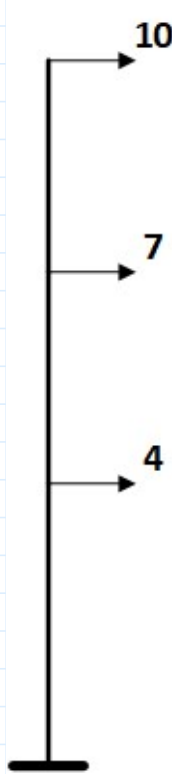
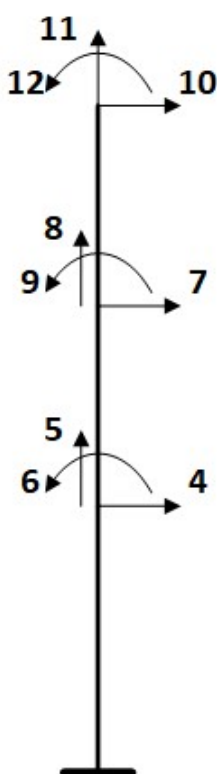
### Statički model

Stepeni slobode: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 i 12



### Dinamički model

Stepeni slobode: 4, 7 i 10



## Matrica krutosti i matrica transformacije

Štap tipa  $k$  (funkcija za matricu krutosti i matricu transformacije)

$$m_{k_k}(E, L, A, I) := \frac{E \cdot I}{L^3} \cdot \begin{bmatrix} \frac{A \cdot L^2}{I} & 0 & 0 & -\frac{A \cdot L^2}{I} & 0 & 0 \\ 0 & 12 & 6 \cdot L & 0 & -12 & 6 \cdot L \\ 0 & 6 \cdot L & 4 \cdot L^2 & 0 & -6 \cdot L & 2 \cdot L^2 \\ -\frac{A \cdot L^2}{I} & 0 & 0 & \frac{A \cdot L^2}{I} & 0 & 0 \\ 0 & -12 & -6 \cdot L & 0 & 12 & -6 \cdot L \\ 0 & 6 \cdot L & 2 \cdot L^2 & 0 & -6 \cdot L & 4 \cdot L^2 \end{bmatrix}$$

$$T_k(\alpha) := \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 & 0 & 0 & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\alpha) & \sin(\alpha) & 0 \\ 0 & 0 & 0 & -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

### Štap 1

$$L_1 := h_1 \quad T_1 := T_k(90 \text{ deg}) \quad m_{k_1\_LKS} := m_{k_k}(E, L_1, A, I) \quad m_{k_1\_GKS} := T_1^T \cdot m_{k_1\_LKS} \cdot T_1$$

	1	2	3	4	5	6	
--	---	---	---	---	---	---	--

$$m_{k_1\_GKS} = \begin{bmatrix} 154687.5 & 0.0 & -309375.0 & -154687.5 & 0.0 & -309375.0 \\ 0.0 & 2475000.0 & 0.0 & 0.0 & -2475000.0 & 0.0 \\ -309375.0 & 0.0 & 825000.0 & 309375.0 & 0.0 & 412500.0 \\ -154687.5 & 0.0 & 309375.0 & 154687.5 & 0.0 & 309375.0 \\ 0.0 & -2475000.0 & 0.0 & 0.0 & 2475000.0 & 0.0 \\ -309375.0 & 0.0 & 412500.0 & 309375.0 & 0.0 & 825000.0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}$$

Vektor stepeni slobode:  $ss_1 := [1 \ 2 \ 3 \ 4 \ 5 \ 6]^T$

### Štap 2

$$L_2 := h_2 \quad T_2 := T_k(90 \text{ deg}) \quad m_{k_2\_LKS} := m_{k_k}(E, L_2, A, I) \quad m_{k_2\_GKS} := T_2^T \cdot m_{k_2\_LKS} \cdot T_2$$

	4	5	6	7	8	9	
--	---	---	---	---	---	---	--

$$m_{k_2\_GKS} = \begin{bmatrix} 366666.7 & 0.0 & -550000.0 & -366666.7 & 0.0 & -550000.0 \\ 0.0 & 3300000.0 & 0.0 & 0.0 & -3300000.0 & 0.0 \\ -550000.0 & 0.0 & 1100000.0 & 550000.0 & 0.0 & 550000.0 \\ -366666.7 & 0.0 & 550000.0 & 366666.7 & 0.0 & 550000.0 \\ 0.0 & -3300000.0 & 0.0 & 0.0 & 3300000.0 & 0.0 \\ -550000.0 & 0.0 & 550000.0 & 550000.0 & 0.0 & 1100000.0 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$$

Vektor stepeni slobode:  $ss_2 := [4 \ 5 \ 6 \ 7 \ 8 \ 9]^T$

### Štap 3

$$L_3 := h_3 \quad T_3 := T_k(90 \text{ deg}) \quad m_{k\_3\_LKS} := m_{k\_k}(E, L_3, A, I) \quad m_{k\_3\_GKS} := T_3^T \cdot m_{k\_3\_LKS} \cdot T_3$$

$$m_{k\_3\_LKS} = \begin{matrix} & \begin{matrix} 7 & 8 & 9 & 10 & 11 & 12 \end{matrix} \\ \begin{matrix} 3300000.0 & 0.0 & 0.0 & -3300000.0 & 0.0 & 0.0 \\ 0.0 & 366666.7 & 550000.0 & 0.0 & -366666.7 & 550000.0 \\ 0.0 & 550000.0 & 1100000.0 & 0.0 & -550000.0 & 550000.0 \\ -3300000.0 & 0.0 & 0.0 & 3300000.0 & 0.0 & 0.0 \\ 0.0 & -366666.7 & -550000.0 & 0.0 & 366666.7 & -550000.0 \\ 0.0 & 550000.0 & 550000.0 & 0.0 & -550000.0 & 1100000.0 \end{matrix} & \begin{bmatrix} 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{bmatrix} \end{matrix}$$

Vektor stepeni slobode:  $ss_3 := [7 \ 8 \ 9 \ 10 \ 11 \ 12]^T$

### Kondenzovana matrica krutosti

Pasivni stepeni slobode: 1, 2 i 3      Aktivni stepeni slobode: 4, 5, 6, 7, 8, 9, 10, 11 i 12

Bitni stepeni slobode: 4, 7 i 10       $b_s := [4 \ 7 \ 10]^T$

Nebitni stepeni slobode: 5, 6, 8, 9, 11 i 12       $n_s := [5 \ 6 \ 8 \ 9 \ 11 \ 12]^T$

$$niz_{ss} := \begin{bmatrix} ss_1 \\ ss_2 \\ ss_3 \end{bmatrix} \quad niz_{mk} := \begin{bmatrix} m_{k\_1\_GKS} \\ m_{k\_2\_GKS} \\ m_{k\_3\_GKS} \end{bmatrix}$$

Funkcija za formiranje matrice sistema povezanih štapova koristeći brojeve stepeni slobode štapova i matrice krutosti štapova

```

K(nizss, nizmk, ssv, ssk) :=
    Klast(ssv), last(ssk) ← 0
    for m ∈ 0..last(nizss)
        SSstap ← nizssm
        for i ∈ 0..last(SSstap)
            for j ∈ 0..last(ssv)
                if SSstapi = ssvj
                    for k ∈ 0..last(SSstap)
                        for l ∈ 0..last(ssk)
                            if SSstapk = sskl
                                MK ← nizmkm
                                Kj,l ← Kj,l + MKi,k
    return K

```

$$Kbb := K(niz_{ss}, niz_{mk}, b_s, b_s)$$

$$Kbn := K(niz_{ss}, niz_{mk}, b_s, n_s)$$

$$Knb := K(niz_{ss}, niz_{mk}, n_s, b_s)$$

$$Knn := K(niz_{ss}, niz_{mk}, n_s, n_s)$$

$$Kbb = \begin{bmatrix} 4 & 7 & 10 \\ 521354.2 & -366666.7 & 0.0 \\ -366666.7 & 733333.3 & -366666.7 \\ 0.0 & -366666.7 & 366666.7 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix}$$

$$Kbn = \begin{bmatrix} 5 & 6 & 8 & 9 & 11 & 12 \\ 0.0 & -240625.0 & 0.0 & -550000.0 & 0.0 & 0.0 \\ 0.0 & 550000.0 & 0.0 & 0.0 & 0.0 & -550000.0 \\ 0.0 & 0.0 & 0.0 & 550000.0 & 0.0 & 550000.0 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix}$$

$$Knb = \begin{bmatrix} 4 & 7 & 10 \\ 0.0 & 0.0 & 0.0 \\ -240625.0 & 550000.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ -550000.0 & 0.0 & 550000.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & -550000.0 & 550000.0 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 8 \\ 9 \\ 11 \\ 12 \end{bmatrix} \quad \text{if } (Knb = Kbn^T, \text{"OK"}, \text{"NO OK"}) = \text{"OK"}$$

$$Knn = \begin{bmatrix} 5 & 6 & 8 & 9 & 11 & 12 \\ 5775000.0 & 0.0 & -3300000.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1925000.0 & 0.0 & 550000.0 & 0.0 & 0.0 \\ -3300000.0 & 0.0 & 6600000.0 & 0.0 & -3300000.0 & 0.0 \\ 0.0 & 550000.0 & 0.0 & 2200000.0 & 0.0 & 550000.0 \\ 0.0 & 0.0 & -3300000.0 & 0.0 & 3300000.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 550000.0 & 0.0 & 1100000.0 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 8 \\ 9 \\ 11 \\ 12 \end{bmatrix}$$

$$Kc := Kbb - Kbn \cdot Knn^{-1} \cdot Knb$$

$$Kc = \begin{bmatrix} 4 & 7 & 10 \\ 360269.1 & -265833.3 & 74861.1 \\ -265833.3 & 293333.3 & -110000.0 \\ 74861.1 & -110000.0 & 48888.9 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix}$$

## Matrica mase

$$Mc := \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = \begin{bmatrix} 4 & 7 & 10 \\ 40.0 & 0.0 & 0.0 \\ 0.0 & 40.0 & 0.0 \\ 0.0 & 0.0 & 30.0 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 10 \end{bmatrix}$$

## Periodi svojstvenih vibracija

$$Trac := \frac{2 \cdot \pi}{\sqrt{\text{eigenvals}(Mc^{-1} \cdot Kc)}} = \begin{bmatrix} 0.04991 \\ 0.13817 \\ 0.84654 \end{bmatrix}$$

$$T_{ton\_1} := Trac_2 = 0.84654$$

$$T_{ton\_2} := Trac_1 = 0.13817$$

$$T_{ton\_3} := Trac_0 = 0.04991$$

## Oblici svojstvenih vibracija

$$Arac := \text{eigenvecs}(Mc^{-1} \cdot Kc) = \begin{bmatrix} -0.70651 & 0.63972 & 0.1884 \\ 0.64535 & 0.50467 & 0.49297 \\ -0.29045 & -0.57972 & 0.8494 \end{bmatrix}$$

$$\text{Ton 1: } Arac\_1 := Arac^{(2)} = \begin{bmatrix} 0.1884 \\ 0.49297 \\ 0.8494 \end{bmatrix} \quad \text{Ton 2: } Arac\_2 := Arac^{(1)} = \begin{bmatrix} 0.63972 \\ 0.50467 \\ -0.57972 \end{bmatrix}$$

$$\text{Ton 3: } Arac\_3 := -Arac^{(0)} = \begin{bmatrix} 0.70651 \\ -0.64535 \\ 0.29045 \end{bmatrix}$$

## Ortonormiranje svojstvenih oblika

Ton 1

$$Mgen\_1 := Arac\_1^T \cdot Mc \cdot Arac\_1 = 32.78514 \quad \phi_1 := \frac{1}{\sqrt{Mgen\_1}} \cdot Arac\_1 = \begin{bmatrix} 0.03290 \\ 0.08610 \\ 0.14835 \end{bmatrix}$$

$$M_1 := \phi_1^T \cdot Mc \cdot \phi_1 = 1.00000$$

Ton 2

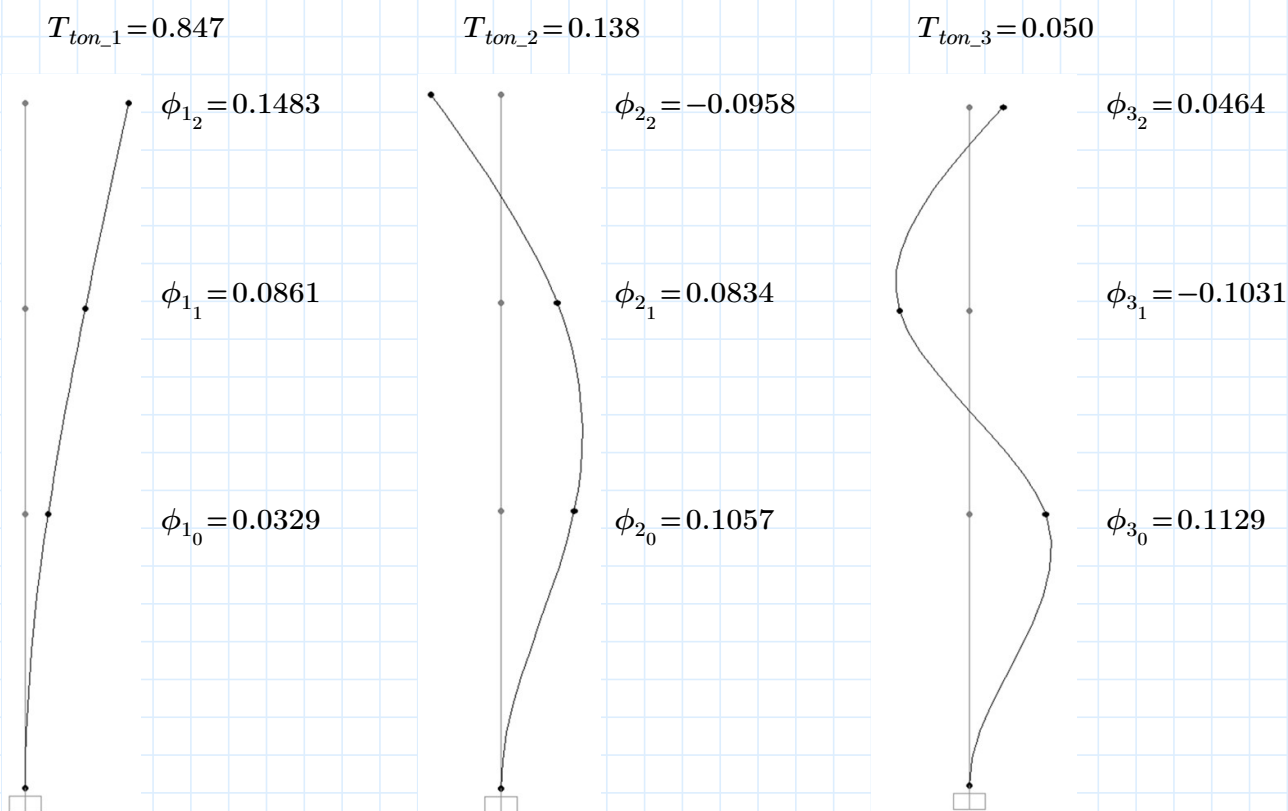
$$Mgen\_2 := Arac\_2^T \cdot Mc \cdot Arac\_2 = 36.63930 \quad \phi_2 := \frac{1}{\sqrt{Mgen\_2}} \cdot Arac\_2 = \begin{bmatrix} 0.10569 \\ 0.08337 \\ -0.09577 \end{bmatrix}$$

$$M_2 := \phi_2^T \cdot Mc \cdot \phi_2 = 1.00000$$

Ton 3

$$Mgen\_3 := Arac\_3^T \cdot Mc \cdot Arac\_3 = 39.15640 \quad \phi_3 := \frac{1}{\sqrt{Mgen\_3}} \cdot Arac\_3 = \begin{bmatrix} 0.11291 \\ -0.10313 \\ 0.04642 \end{bmatrix}$$

$$M_3 := \phi_3^T \cdot Mc \cdot \phi_3 = 1.00000$$



## Elastični i projektni spektar odgovora za horizontalan pravac

### Tip 1 elastičnog odgovora

```
parametri_tip_1 (tipTla) := || if tipTla = "A"
                             || [1.00 0.15 0.4 2.0]
                             || if tipTla = "B"
                             || [1.20 0.15 0.5 2.0]
                             || if tipTla = "C"
                             || [1.15 0.20 0.6 2.0]
                             || if tipTla = "D"
                             || [1.35 0.20 0.8 2.0]
                             || if tipTla = "E"
                             || [1.40 0.15 0.5 2.0]
```

```
niz := parametri_tip_1 (tipTla) = [1.15 0.2 0.6 2]
```

$Ss := niz_{0,0} = 1.15$

$Tb := niz_{0,1} = 0.2$

$Tc := niz_{0,2} = 0.6$

$Td := niz_{0,3} = 2$

$$Se(T_{ton}) := \begin{cases} \text{if } 0 \leq T_{ton} \leq Tb \\ \quad \left\| a_g \cdot Ss \cdot \left( 1 + \frac{T_{ton}}{Tb} \cdot (\eta \cdot 2.5 - 1) \right) \right\| \\ \text{if } Tb < T_{ton} \leq Tc \\ \quad \left\| a_g \cdot Ss \cdot \eta \cdot 2.5 \right\| \\ \text{if } Tc < T_{ton} \leq Td \\ \quad \left\| a_g \cdot Ss \cdot \eta \cdot 2.5 \cdot \frac{Tc}{T_{ton}} \right\| \\ \text{if } Td < T_{ton} \leq 4 \\ \quad \left\| a_g \cdot Ss \cdot \eta \cdot 2.5 \cdot \frac{Tc \cdot Td}{T_{ton}^2} \right\| \end{cases}$$

$$Sd(T_{ton}, q) := \begin{cases} \text{if } 0 \leq T_{ton} \leq Tb \\ \quad \left\| a_g \cdot Ss \cdot \left( \frac{2}{3} + \frac{T_{ton}}{Tb} \cdot \left( \frac{2.5}{q} - \frac{2}{3} \right) \right) \right\| \\ \text{if } Tb < T_{ton} \leq Tc \\ \quad \left\| a_g \cdot Ss \cdot \frac{2.5}{q} \right\| \\ \text{if } Tc < T_{ton} \leq Td \\ \quad \left\| a_g \cdot Ss \cdot \frac{2.5}{q} \cdot \frac{Tc}{T_{ton}} \right\| \\ \text{if } Td < T_{ton} \\ \quad \left\| a_g \cdot Ss \cdot \frac{2.5}{q} \cdot \frac{Tc \cdot Td}{T_{ton}^2} \right\| \end{cases}$$

**Projektno spektralno ubrzanje**

$$Sa11 := Sd(T_{ton\_1}, q) = 1.333$$

$$Sa22 := Sd(T_{ton\_2}, q) = 1.531$$

$$Sa33 := Sd(T_{ton\_3}, q) = 1.034$$

$$\beta := 0.2$$

$$\beta \cdot a_g = 0.196$$

Ograničenje projektnog spektralnog ubrzanja:

$$Sa(Spa) := \begin{cases} \text{if } Spa < \beta \cdot a_g \\ \quad \left\| \text{return } \beta \cdot a_g \right\| \\ \text{else} \\ \quad \left\| \text{return } Spa \right\| \end{cases}$$

$$Sa1 := Sa(Sa11) = 1.333$$

$$Sa2 := Sa(Sa22) = 1.531$$

$$Sa3 := Sa(Sa33) = 1.034$$

**Faktor "participacije"**

Vektor uticajnih koeficijenata:  $s := \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$\Gamma_1 := \frac{\phi_1^T \cdot Mc \cdot s}{M_1} = 9.210$$

$$\Gamma_2 := \frac{\phi_2^T \cdot Mc \cdot s}{M_2} = 4.689$$

$$\Gamma_3 := \frac{\phi_3^T \cdot Mc \cdot s}{M_3} = 1.783$$

**Efektivna masa**

Ukupna masa dinamičkog modela:

$$Mukupno := \begin{cases} suma \leftarrow 0 \\ \text{for } i \in 0 \dots \text{last}(Mc^{(0)}) \\ \quad \left\| suma \leftarrow suma + Mc_{i,i} \right\| \\ \text{return } suma \end{cases} = 110.0$$

Efektivna masa za T1:

$$L1 := \phi_1^T \cdot Mc \cdot s = 9.210$$

$$Mef\_1 := \frac{L1^2}{M_1} = 84.83$$

$$\frac{Mef\_1}{Mukupno} = 0.771$$

Efektivna masa za T2:

$$L2 := \phi_2^T \cdot Mc \cdot s = 4.689 \quad Mef\_2 := \frac{L2^2}{M_2} = 21.99 \quad \frac{Mef\_2}{Mukupno} = 0.200$$

Efektivna masa za T3:

$$L3 := \phi_3^T \cdot Mc \cdot s = 1.783 \quad Mef\_3 := \frac{L3^2}{M_3} = 3.18 \quad \frac{Mef\_3}{Mukupno} = 0.029$$

$$Mef\_1 + Mef\_2 + Mef\_3 = 110.0$$

$$\frac{Mef\_1 + Mef\_2 + Mef\_3}{Mukupno} = 1$$

## Seizmičke sile

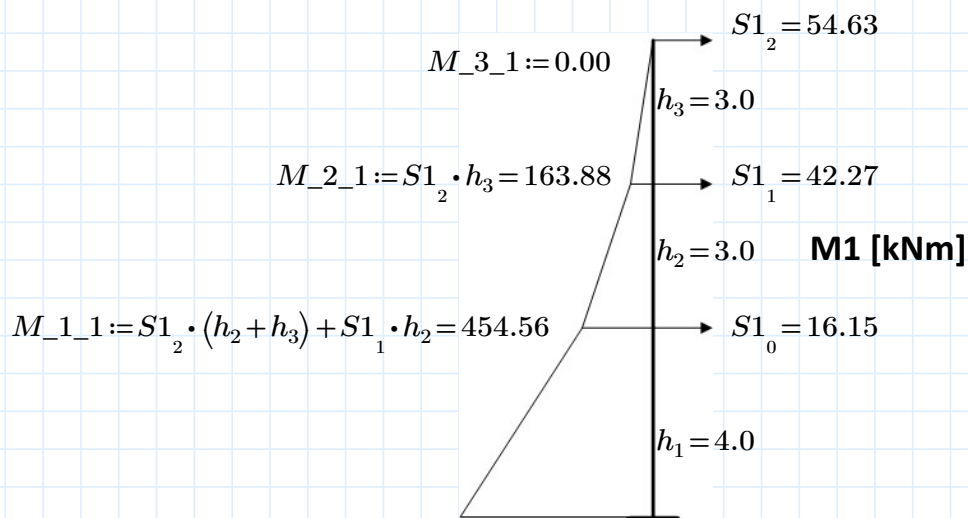
$$\text{Ton 1: } S1 := \Gamma_1 \cdot Mc \cdot \phi_1 \cdot Sa1 = \begin{bmatrix} 16.15492 \\ 42.27061 \\ 54.62535 \end{bmatrix} \quad \text{Ton 2: } S2 := \Gamma_2 \cdot Mc \cdot \phi_2 \cdot Sa2 = \begin{bmatrix} 30.35893 \\ 23.94970 \\ -20.63348 \end{bmatrix}$$

$$\text{Ton 3: } S3 := \Gamma_3 \cdot Mc \cdot \phi_3 \cdot Sa3 = \begin{bmatrix} 8.32540 \\ -7.60471 \\ 2.56694 \end{bmatrix}$$

## Dijagram momenata savijanja (SRSS pravilo)

**Ton 1**

$$S1 = \begin{bmatrix} 16.15 \\ 42.27 \\ 54.63 \end{bmatrix}$$

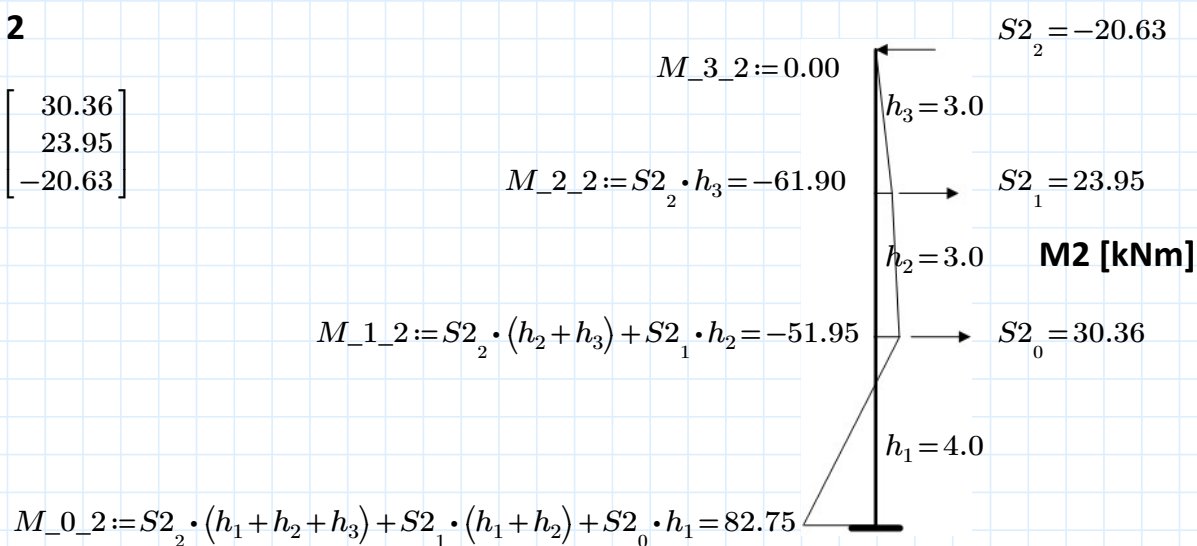


$$M_{0\_1} := S1_2 \cdot (h_1 + h_2 + h_3) + S1_1 \cdot (h_1 + h_2) + S1_0 \cdot h_1 = 906.77$$



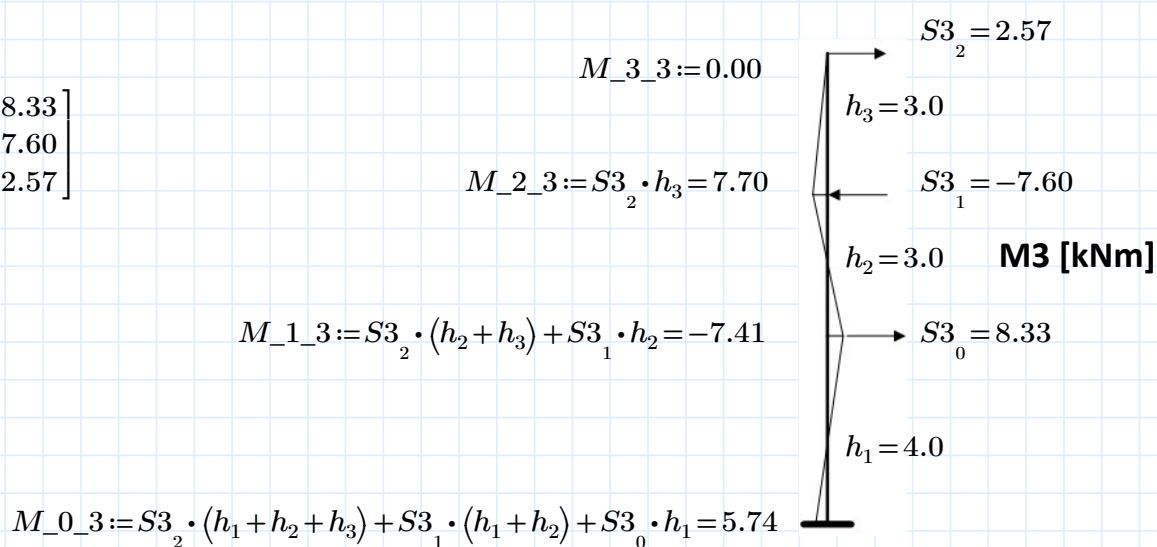
## Ton 2

$$S2 = \begin{bmatrix} 30.36 \\ 23.95 \\ -20.63 \end{bmatrix}$$

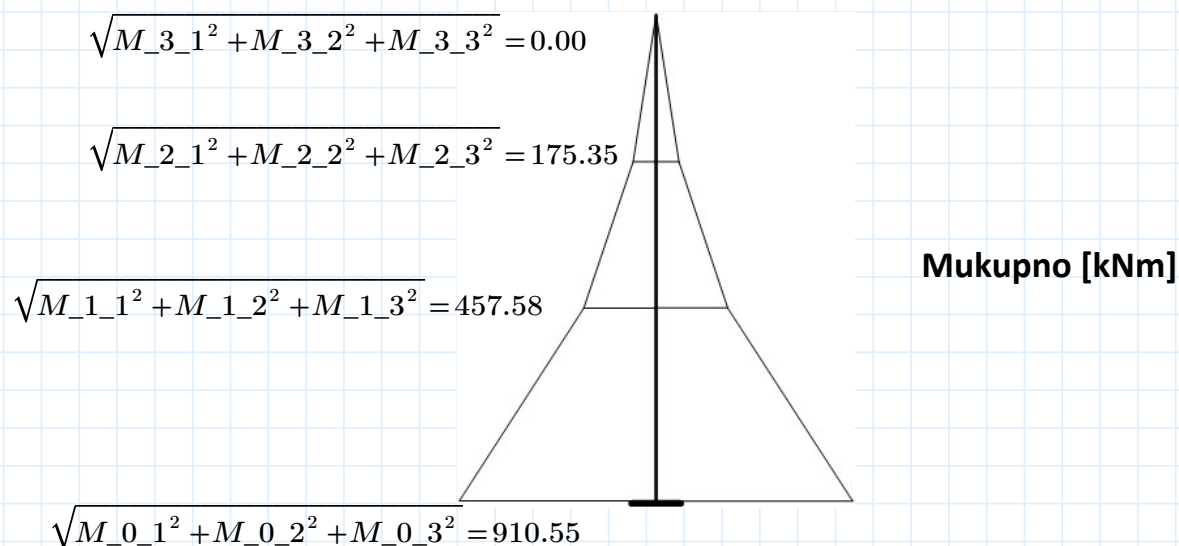


## Ton 3

$$S3 = \begin{bmatrix} 8.33 \\ -7.60 \\ 2.57 \end{bmatrix}$$



## Ukupni dijagram M (SRSS pravilo)



## Relativno horizontalno pomeranje u nivou masa (SRSS pravilo)

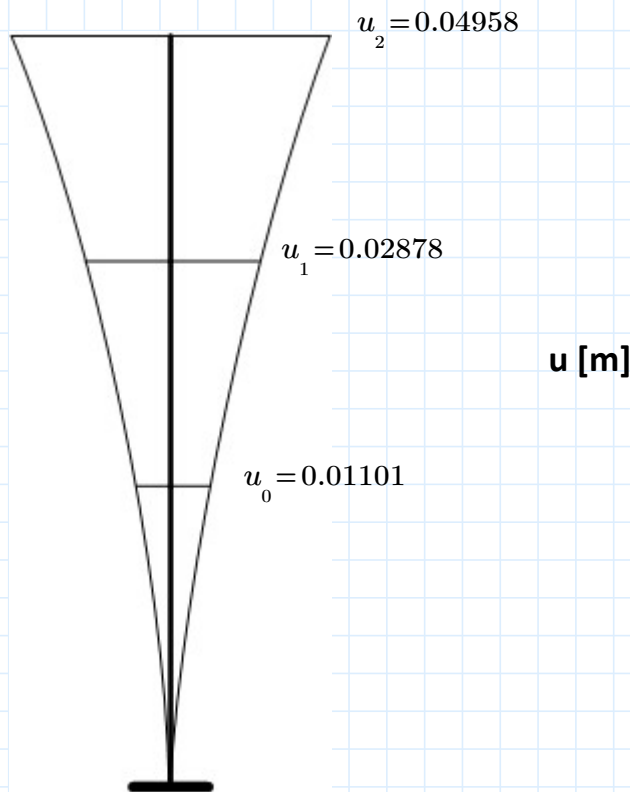
Određeno na osnovu elastičnog spektra odgovora

$$u1 := \Gamma_1 \cdot \phi_1 \cdot \frac{Se(T_{ton\_1})}{\left(\frac{2 \cdot \pi}{T_{ton\_1}}\right)^2} = \begin{bmatrix} 0.01100 \\ 0.02877 \\ 0.04958 \end{bmatrix}$$

$$u2 := \Gamma_2 \cdot \phi_2 \cdot \frac{Se(T_{ton\_2})}{\left(\frac{2 \cdot \pi}{T_{ton\_2}}\right)^2} = \begin{bmatrix} 0.00055 \\ 0.00043 \\ -0.00050 \end{bmatrix}$$

$$u3 := \Gamma_3 \cdot \phi_3 \cdot \frac{Se(T_{ton\_3})}{\left(\frac{2 \cdot \pi}{T_{ton\_3}}\right)^2} = \begin{bmatrix} 0.00002 \\ -0.00002 \\ 0.00001 \end{bmatrix}$$

$$u := \begin{bmatrix} \sqrt{(u1_0)^2 + (u2_0)^2 + (u3_0)^2} \\ \sqrt{(u1_1)^2 + (u2_1)^2 + (u3_1)^2} \\ \sqrt{(u1_2)^2 + (u2_2)^2 + (u3_2)^2} \end{bmatrix} = \begin{bmatrix} 0.01101 \\ 0.02878 \\ 0.04958 \end{bmatrix}$$



Određeno na osnovu projektnog spektra odgovora

$$u1 := \Gamma_1 \cdot \phi_1 \cdot \frac{Sd(T_{ton\_1}, q)}{\left(\frac{2 \cdot \pi}{T_{ton\_1}}\right)^2} \cdot q = \begin{bmatrix} 0.01100 \\ 0.02877 \\ 0.04958 \end{bmatrix}$$

$$u2 := \Gamma_2 \cdot \phi_2 \cdot \frac{Sd(T_{ton\_2}, q)}{\left(\frac{2 \cdot \pi}{T_{ton\_2}}\right)^2} \cdot q = \begin{bmatrix} 0.00055 \\ 0.00043 \\ -0.00050 \end{bmatrix}$$

$$u3 := \Gamma_3 \cdot \phi_3 \cdot \frac{Sd(T_{ton\_3}, q)}{\left(\frac{2 \cdot \pi}{T_{ton\_3}}\right)^2} \cdot q = \begin{bmatrix} 0.00002 \\ -0.00002 \\ 0.00001 \end{bmatrix}$$

$$u := \begin{bmatrix} \sqrt{(u1_0)^2 + (u2_0)^2 + (u3_0)^2} \\ \sqrt{(u1_1)^2 + (u2_1)^2 + (u3_1)^2} \\ \sqrt{(u1_2)^2 + (u2_2)^2 + (u3_2)^2} \end{bmatrix} = \begin{bmatrix} 0.01101 \\ 0.02878 \\ 0.04958 \end{bmatrix} \text{ [m]}$$